Automated Global Analysis of Experimental Dynamics through Low-Dimensional Linear Embeddings

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Abstract: Dynamical Systems Theory provides a framework for analyzing and predicting the behavior of time-varying systems, with applications across diverse fields such as biology, engineering, and climate science. However, applying the tools from this theoretical framework remains difficult, as real-world systems often exhibit nonlinear, high-dimensional and unknown dynamics. In this work, we present a data-driven computational framework that derives low-dimensional linear embeddings of nonlinear dynamics from raw experimental data. Our framework leverages time-delay embedding, a technique that can capture complex temporal dependencies, and physics-informed deep autoencoders, which incorporate known physical relationships to guide the learning these latent representations. Additionally, we employ an annealing-based regularization to ensure robust and generalizable models by gradually adjusting model parameters during training.

This research builds upon a significant body of work dedicated to uncovering latent structures within experimental data from physical systems. Recent advancements have demonstrated the potential of deep convolutional autoencoders to discover neural state variables directly from video data [1, 2]. Other studies have explored the use of deep learning to find latent linear models that approximate the behavior of nonlinear dynamical systems [4, 3]. Our method produces low-dimensional representations that enable not

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only extended, accurate predictions but also facilitate the intepretability of the system's dynamics. The linear structure of these models can be readily exploited to identify Lyapunov functions and intricate invariant sets, which are pivotal to understanding a system's stability.

By enabling the discovery of low-dimensional linearizations for complex systems, our framework opens new avenues for analyzing dynamical behaviors in fields such as physics, climate science, and engineering. This framework represents a promising advancement in nonlinear system analysis by making complex dynamical behavior accessible through interpretable linear models.

References

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