

VENI, VINDy, VICI: a generative framework for discovering dynamics with uncertainty quantification

Paolo Conti^a, Jonas Kneifl^b, Andrea Manzoni^a, Attilio Frangi^c, Jörg Fehr^b, Steven Brunton^d, Nathan Kutz^e

^aMOX – Dept. of Mathematics and ^cDept. of Civil Engineering, Politecnico di Milano, Milan, Italy

^bInstitute of Engineering and Computational Mechanics, University of Stuttgart, Stuttgart, Germany

^dDept. of Mechanical Engineering and ^eDept. of Applied Mathematics and Electrical and Computer Engineering, University of Washington, Seattle, USA

In the era of big data, advancements in sensor and measurement technologies have enabled frequent access to rich spatio-temporal data. This development has paved the way for a new paradigm in which descriptive and predictive dynamical models are automatically extracted from data streams, minimizing the need for extensive prior knowledge and intrusive modeling.

However, in scientific computing accurate data are often high-dimensional and expensive to obtain. Moreover, in real-life scenarios data are affected by noise and uncertainties. These challenges necessitate a robust approach to modeling that can handle such complexities.

To address this, we present a generative framework for constructing predictive, robust, and reliable reduced-order models. Our approach consists in the discovery of *latent variables* — reduced-dimension variables that effectively describe the dynamics of the phenomenon and are potentially more interpretable — and a *latent dynamical model* in this new set of variables.

Our method¹, termed VENI, VINDy, VICI, comprises three key components. The first, Variational Encoding of Noisy Inputs (VENI), employs variational autoencoders to identify the distribution of reduced variables. Concurrently, we utilize Variational Identification of Nonlinear Dynamics (VINDy), a newly proposed variational version of SINDy³, to learn the distribution of coefficients for a predetermined set of candidate functions. Once trained offline, the model can be queried for new parameter instances or initial conditions to compute full-time solutions. The probabilistic framework enables uncertainty quantification through Variational Inference, naturally providing Certainty Intervals (VICI).

We demonstrate the effectiveness of our method in re-discovering high-dimensional, nonlinear dynamical systems governed by partial differential equations, including applications in structural mechanics and fluid dynamics.

Our work exemplifies the integration of multiple components in scientific discovery, offering a comprehensive system for dynamic model identification that supports robust scientific insights.

REFERENCES

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Paolo Conti Politecnico di Milano, Dept. of Mathematics, Milan, Italy.

Tel: +39 340 6332219 (IT) **E-mail:** paolo.conti@polimi.it