Generative Adversarial Symmetry Discovery

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Generative Adversarial Training Architecture

Existing equivariant neural networks rely on explicit knowledge about symmetry, which is sometimes unavailable. Our work aims to discover unknown symmetry directly from data.

**Invariance, Equivariance and Data Distribution**

Invariance and equivariance have become an important and intuitive bias in deep learning architectures. A function \( f: X \rightarrow Y \) is invariant to a group \( G \) if \( f(g \cdot x) = f(x) \) for all \( g \in G, x \in X \). It is \( G \)-equivariant if \( f(g \cdot x) = g \cdot f(x) \) for \( g \in G, x \in X \). From another perspective, invariant or equivariant transformations preserve the data distribution.\[ p_f(x, y) = p_{g \cdot f}(x, y) \]

**Comparison of Symmetry Discovery Methods**

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<thead>
<tr>
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<tbody>
<tr>
<td>Discrete group</td>
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<td>✓</td>
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<tr>
<td>Continuous group</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Subset of given group</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Subset of unknown group</td>
<td>✗</td>
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Different approaches with deep learning have been developed to discover symmetries from data. Our approach (LieGAN) is the first to address the discovery of such a variety of symmetries including discrete group, continuous group, and subset of given or unknown group.

**Our Contributions**

We propose the framework of Lie algebras Generative Adversarial Network (LieGAN). Combining generative adversarial training and the theory of Lie groups, our model:

- Learns a distribution over symmetry transformations and produces a transformed data distribution that is indistinguishable from original distribution.
- Discovers various general linear symmetries in datasets, including the rotation group \( SO(3) \)^+.\[ \mathbb{R}^3 \rightarrow \mathbb{R}^3 \]
- Can be combined with customized equivariant neural networks to construct arbitrary group equivariant models and achieve excellent performance in predictive tasks.

Our full paper is available at: https://arxiv.org/abs/2302.00286

**Problem**

How to design a data-driven approach to automatically discover symmetries, i.e. invariances and equivariances, in a predictive task?

**Our Substitutions**

**Task #2: Top Quark Tagging**

Task: Binary classification between top quark jets and lighter quarks. The input is the four-momenta of the constituents of the particle jets.

Discovery: LieGAN discovers the symmetry of restricted Lorentz group, \( SO(1, 3)^+ \). It learns the boosts along different spatial dimensions (row 2 of \( \mathbb{R}^3 \)) and the rotations within spatial dimensions (row 3 of \( \mathbb{R}^3 \)).

The figure shows how the data distribution is transformed by one group element sampled from LieGAN.

**References**


Acknowledgements
This work was supported in part by the U.S. Department Of Energy, Office of Science, U.S. Army Research Office under Grant W911NF-20-1-0334, Google Faculty Award, Amazon Research Award, and NSF Grants #2134174, #2107526 and #2134176.