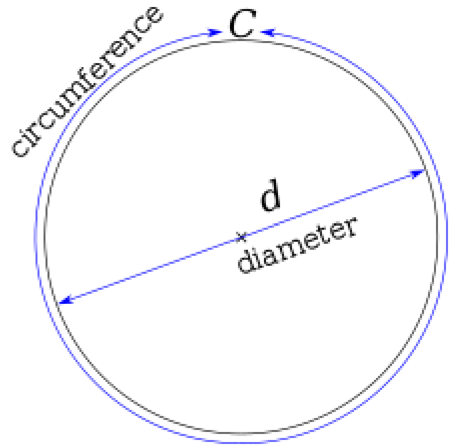
Ramanujan Machine

Generating Conjectures on Mathematical Constants

Itay Beit-Halachmi, Rotem Elimelech, Ofir David, and Ido Kaminer

A tour of the real line…



$\pi = 3.1415\ldots$

$1,2,3,5,8,13,21,34,\ldots$

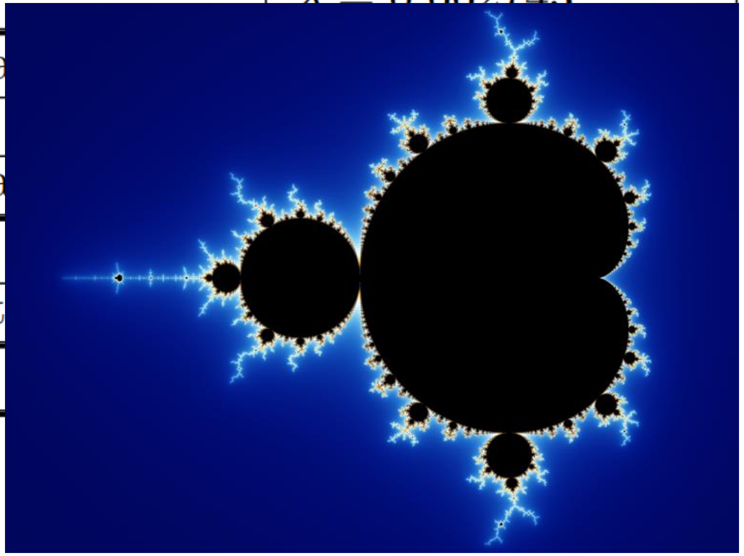
$\phi = 1.618033\ldots$

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

$\zeta(3) = 1.2020569\ldots$

Fundamental constants in many fields of science

Field	Name	Decimal Expansion
Related to Continued Fractions	Lévy's constant	$\gamma = 3.275822\ldots$
	Khinchin's constant	$K_0 = 2.685452\ldots$
Chaos Theory	First Feigenbaum constant	$\delta = 4.669201\ldots$
	Second Feigenbaum constant	$\alpha = 2.502907\ldots$
	Laplace Limit	$\lambda = 0.662743$
Number Theory	Twin Prime consta	
	Meissel – Mertens	
	Landau–Ramanuja	
Combinatorics	Euler–Mascheroni	
	Catalan's constant	
...	...	



Open questions in **chaos theory**

Feigenbaum's constants appear in problems of fluid-flow turbulence, electronic oscillators, chemical reactions, and in the Mandelbrot set

Fundamental constants in many fields of science

Constants provide an **absolute ground truth**, with unlimited amounts of data

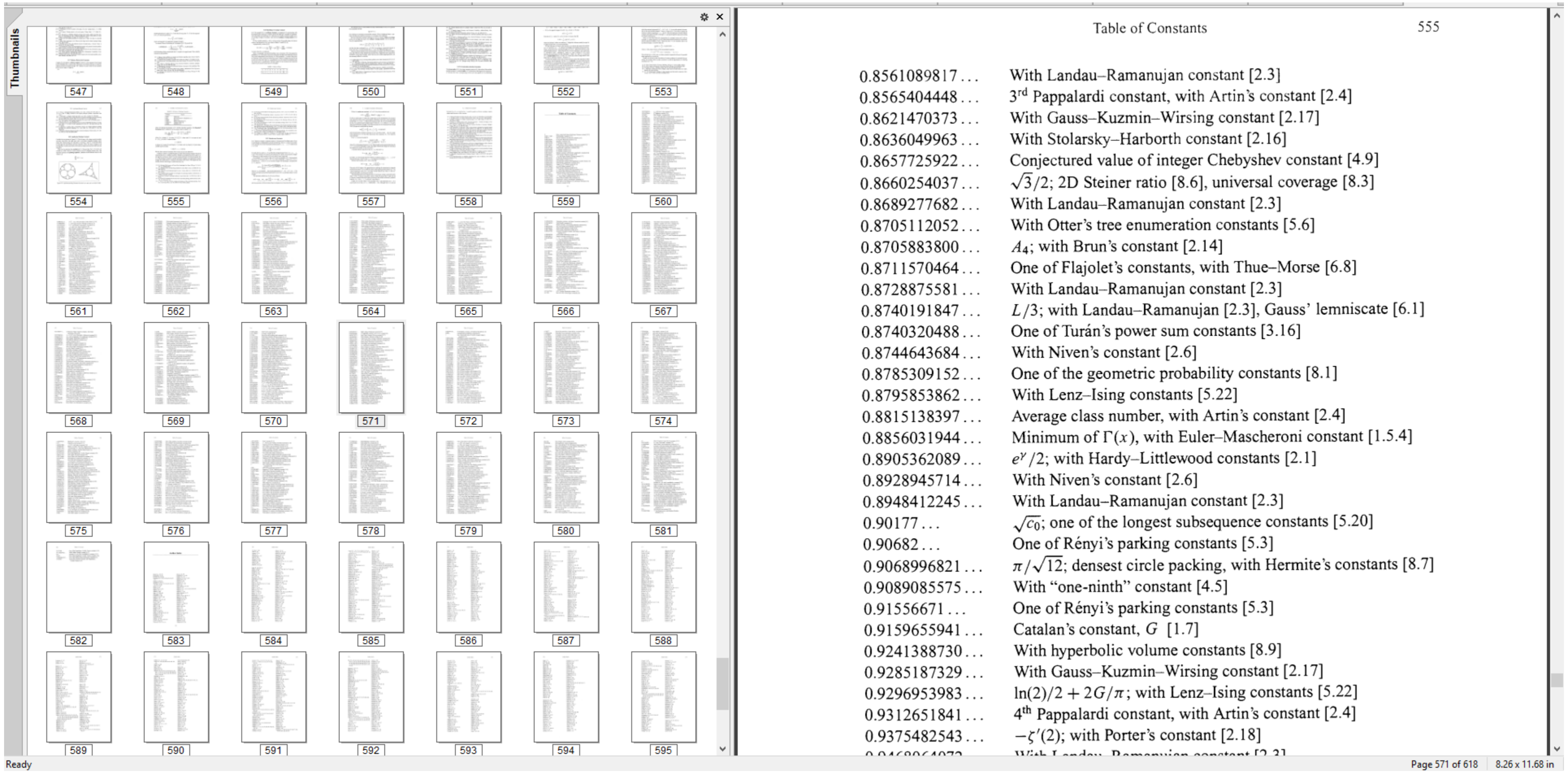


Table of Constants 555

0.8561089817...	With Landau–Ramanujan constant [2.3]
0.8565404448...	3rd Pappalardi constant, with Artin's constant [2.4]
0.8621470373...	With Gauss–Kuzmin–Wirsing constant [2.17]
0.8636049963...	With Stolarsky–Harborth constant [2.16]
0.8657725922...	Conjectured value of integer Chebyshev constant [4.9]
0.8660254037...	$\sqrt{3}/2$; 2D Steiner ratio [8.6], universal coverage [8.3]
0.8689277682...	With Landau–Ramanujan constant [2.3]
0.8705112052...	With Otter's tree enumeration constants [5.6]
0.8705883800...	A_4; with Brun's constant [2.14]
0.8711570464...	One of Flajolet's constants, with Thue–Morse [6.8]
0.8728875581...	With Landau–Ramanujan constant [2.3]
0.8740191847...	$L/3$; with Landau–Ramanujan [2.3], Gauss' lemniscate [6.1]
0.8740320488...	One of Turán's power sum constants [3.16]
0.8744643684...	With Niven's constant [2.6]
0.8785309152...	One of the geometric probability constants [8.1]
0.8795853862...	With Lenz–Ising constants [5.22]
0.8815138397...	Average class number, with Artin's constant [2.4]
0.8856031944...	Minimum of $\Gamma(x)$, with Euler–Mascheroni constant [1.5.4]
0.8905362089...	$e^{\gamma}/2$; with Hardy–Littlewood constants [2.1]
0.8928945714...	With Niven's constant [2.6]
0.8948412245...	With Landau–Ramanujan constant [2.3]
0.90177...	$\sqrt{c_0}$; one of the longest subsequence constants [5.20]
0.90682...	One of Rényi's parking constants [5.3]
0.9068996821...	$\pi/\sqrt{12}$; densest circle packing, with Hermite's constants [8.7]
0.9089085575...	With "one-ninth" constant [4.5]
0.91556671...	One of Rényi's parking constants [5.3]
0.9159655941...	Catalan's constant, G [1.7]
0.9241388730...	With hyperbolic volume constants [8.9]
0.9285187329...	With Gauss–Kuzmin–Wirsing constant [2.17]
0.9296953983...	$\ln(2)/2 + 2G/\pi$; with Lenz–Ising constants [5.22]
0.9312651841...	4th Pappalardi constant, with Artin's constant [2.4]
0.9375482543...	$-\zeta'(2)$; with Porter's constant [2.18]

Mathematical Constants, Steven R. Finch (Cambridge University Press, 2003)
Encyclopedia of Mathematics and its Applications; v. 94

Leonhard Euler

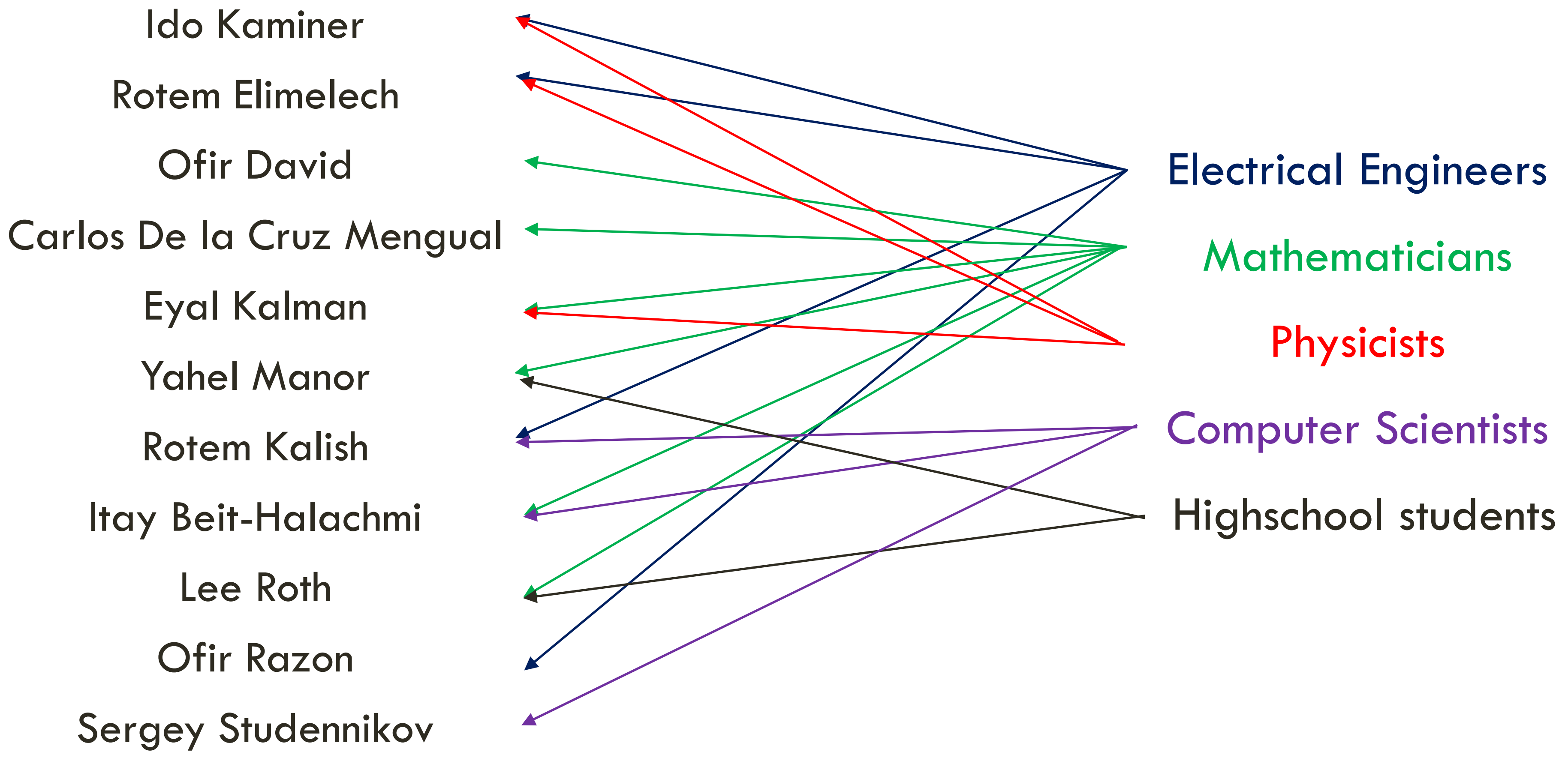
$$\frac{4}{\pi}+1=2+\cfrac{1^2}{2+\cfrac{3^2}{2+\cfrac{5^2}{2+\cdots}}}$$

Srinivasa Ramanujan

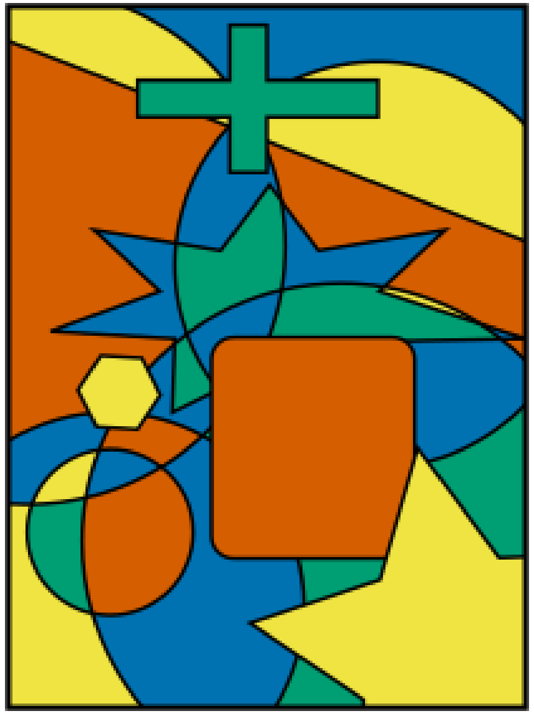
$$e^{\frac{2\pi}{8}}\sqrt{\frac{\sqrt{2}-1}{2}}=\cfrac{1}{1+\cfrac{e^{-2\pi}}{1+e^{-2\pi}+\cfrac{e^{-4\pi}}{1+e^{-4\pi}+\cdots}}}$$

- **The Goal: Automatic Generation of conjectures on Fundamental Constants**
- Algorithms
- Distributed Computing Community
- Mathematical Discoveries
- Future Developments

The Team


Ido Kaminer
Rotem Elimelech
Ofir David
Carlos De la Cruz Mengual
Eyal Kalman
Yahel Manor
Rotem Kalish
Itay Beit-Halachmi
Lee Roth
Ofir Razon
Sergey Studennikov
Electrical Engineers
Mathematicians
Physicists
Computer Scientists
Highschool students

Automated Theorem Proving



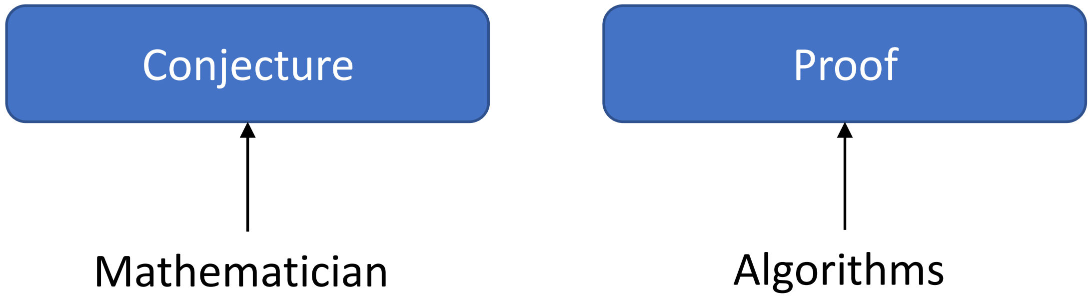
The Four Color Theorem: No more than four colors are required to color the regions of any map, so that no two adjacent regions have the same color.

Conjectured by F. Gurthrie in 1852, proven by Appel and Haken in 1977 using computers.

Conjecture ← Mathematician

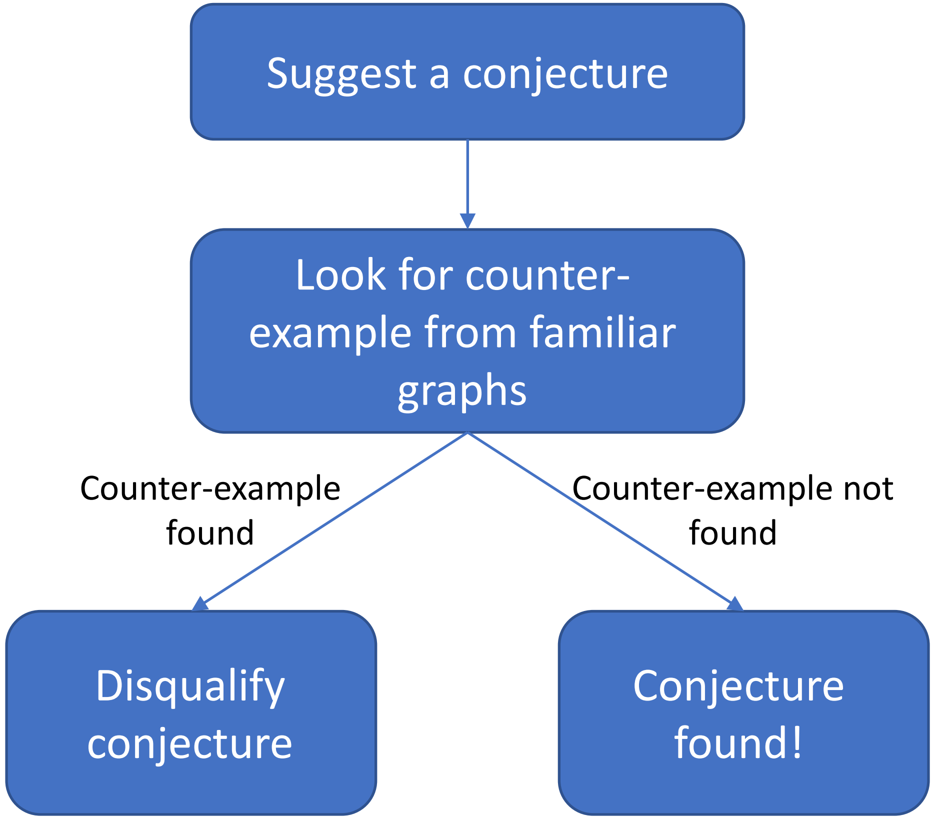
Proof ← Algorithms

Automated Conjecture Generation



Automated Conjecture Generation

Graffiti, Fajtlowicz, 1985



Over 60 publications regarding suggested conjectures!

PSLQ, Ferguson and Bailey 1992

Input $x_i \in \mathbb{R}$

$$a_1x_1 + a_2x_2 + a_3x_3 \ldots = 0$$

Output $a_i \in \mathbb{Z}$

Countless discoveries, among them:

$$\pi = \sum_{n=0}^{\infty} \frac{1}{16^n}\left(\frac{4}{8n+1} - \frac{4}{8n+4} - \frac{4}{8n+5} - \frac{4}{8n+6}\right)$$

Compute base-16 digits of π starting at any given position

- The Goal: Automatic Generation of conjectures on Fundamental Constants
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$$\frac{4}{\pi - 2} = 3 + \cfrac{1 \cdot 3}{5 + \cfrac{2 \cdot 4}{7 + \cfrac{3 \cdot 5}{9 + \cfrac{4 \cdot 6}{11 + \cfrac{5 \cdot 7}{13 + \cfrac{6 \cdot 8}{\dots}}}}}}$$

Convergence	Known / New	Formula	Polynomials
Exponential	known	$\frac{4}{\pi} = 1 + \frac{1}{3+\frac{4}{5+\frac{9}{7+\dots}}}$	$a_n = 1 + 2n,\ b_n = n^2$
Super-Exponential	new and proven	$e = 3 + \frac{-1}{4+\frac{-2}{5+\frac{-3}{6+\dots}}}$	$a_n = 3 + n,\ b_n = -n$

$$\tan 1 = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{\dots}}}}}}$$

Raayoni, G., Gottlieb, S., Manor, Y. et al. *Nature* **590,** 67–73 (2021)

Razon, O., et al. *arXiv*:2212.09470 (2022)

Meet in the Middle with Regular Formulas

	LHS	RHS
e	$2.718281\ldots$	$3.1423\ldots$
$e+1$	$3.718281\ldots$	$2.7183\ldots$
$\vdots$	$17.543901\ldots$	$6.4123\ldots$
	$0.748123\ldots$	$17.5348\ldots$
	$0.412318\ldots$	$0.7841\ldots$

$\overset{!}{}$ $\overset{?}{=}$

$$3+\cfrac{-1}{4+\cfrac{-2}{5+\cfrac{-3}{\ddots}}}$$

Re-check at higher precision…

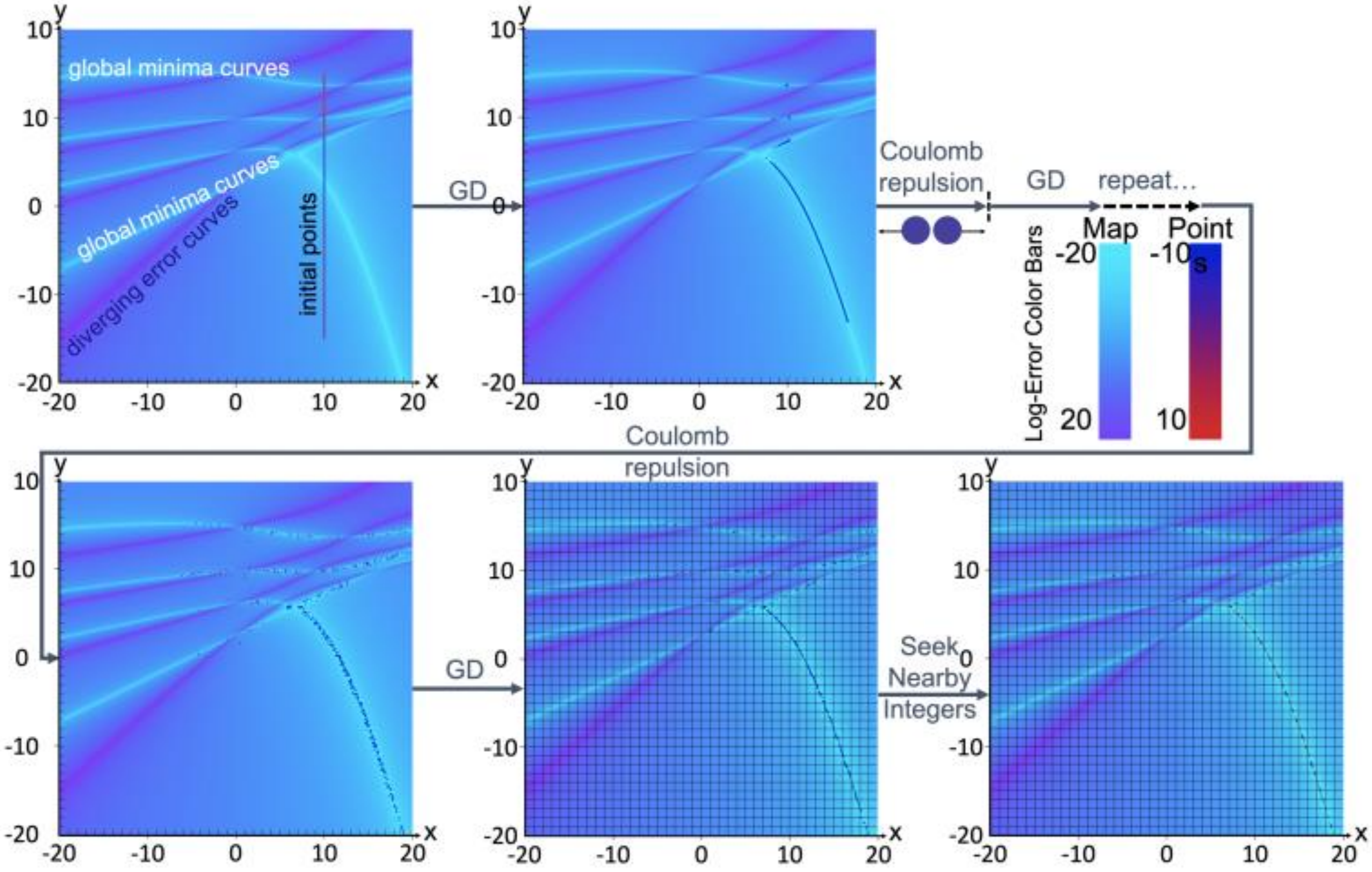
$$2.71828182845\ldots \overset{!!}{=} 2.71828182845\ldots$$

If re-check is successful, output a conjecture!

$$e = 3+\cfrac{-1}{4+\cfrac{-2}{5+\cfrac{-3}{\ddots}}}$$

Raayoni, G., Gottlieb, S., Manor, Y. et al. *Nature* **590,** 67–73 (2021)

Descent&Repel



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ESMA

$$\frac{e^2+1}{2e^2-2} = 1 + \cfrac{-1}{2 + \cfrac{1}{1 + \cfrac{1}{11 + \cfrac{-1}{1 + \cfrac{1}{\ddots}}}}}$$

Pattern?

If the pattern found is simple enough, and retesting at high precision maintains equality, then output a conjecture!

Razon, O., et al. *arXiv*:2212.09470 (2022)

More results…

Novelty	Formula
known	$-1+e=1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cdots}}}$
new and unproven	$\frac{1+e}{4(-1+e)}=1-\cfrac{1}{2+\cfrac{1}{5+\cfrac{1}{2-\cdots}}}$
known	$\frac{1+e}{-1+e}=2+\cfrac{1}{6+\cfrac{1}{10+\cfrac{1}{14+\cdots}}}$
new and unproven	$\frac{-5+3e}{3-e}=12-\cfrac{1}{1+\cfrac{1}{5-\cfrac{1}{1+\cdots}}}$
new and unproven	$\frac{2+2e}{-1+3e}=2-\cfrac{1}{1+\cfrac{1}{24+\cfrac{1}{3-\cdots}}}$
new and unproven	$\frac{-3+5e}{-6+6e}=1+\cfrac{1}{36+\cfrac{1}{2-\cfrac{1}{4-\cdots}}}$
new and unproven	$\frac{1}{-2+2e^2}=1-\cfrac{1}{1+\cfrac{1}{11+\cfrac{1}{2-\cdots}}}$

Novelty	Formula
known	$\tan(1)=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cdots}}}$
new and unproven	$\frac{\tan(1)}{-1+\tan(1)}=3-\cfrac{1}{5-\cfrac{1}{7-\cfrac{1}{9-\cdots}}}$
new and unproven	$\frac{2-\tan(1)}{-1+\tan(1)}=1-\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{1+\cdots}}}$
new and unproven	$\frac{2}{\tan(1)}=2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{3-\cdots}}}$
new and unproven	$\frac{1}{-2+2\tan(1)}=1-\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{3-\cdots}}}$
new and unproven	$\frac{-2+2\tan(1)}{-3+2\tan(1)}=10-\cfrac{1}{4-\cfrac{1}{2-\cfrac{1}{5-\cdots}}}$
new and unproven	$\frac{-5+4\tan(1)}{-7+5\tan(1)}=2-\cfrac{1}{3-\cfrac{1}{2-\cfrac{1}{2-\cdots}}}$

Novelty	Formula
known	$\frac{J_0(1)}{J_1(1)}=2-\cfrac{1}{4-\cfrac{1}{6-\cfrac{1}{8-\cdots}}}$
new and unproven	$-1+\frac{J_0(1)}{J_1(1)}=1-\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{5-\cdots}}}$
new and unproven	$\frac{\frac{-3J_1(1)}{2}+J_0(1)}{-J_0(1)+2J_1(1)}=1-\cfrac{1}{11+\cfrac{1}{1+\cfrac{1}{3-\cdots}}}$
new and unproven	$\frac{J_0(1)}{2J_1(1)}=1-\cfrac{1}{8-\cfrac{1}{3-\cfrac{1}{16-\cdots}}}$
new and unproven	$\frac{-J_0(1)+3J_1(1)}{-J_1(1)+J_0(1)}=1+\cfrac{1}{2-\cfrac{1}{1+\cfrac{1}{1+\cdots}}}$
new and unproven	$\frac{-2J_0(1)+4J_1(1)}{-4J_0(1)+7J_1(1)}=12-\cfrac{1}{4-\cfrac{1}{20-\cfrac{1}{6-\cdots}}}$
new and unproven	$\frac{-12047J_1(1)+6928J_0(1)}{8(1777J_0(1)+3090J_1(1))}=1-\cfrac{1}{112-\cfrac{1}{2-\cfrac{1}{143+\cdots}}}$
known	$\frac{J_1(1)}{J_2(1)}=4-\cfrac{1}{6-\cfrac{1}{8-\cfrac{1}{10-\cdots}}}$
new and unproven	$\frac{J_0(1)}{J_2(1)}=6+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}$

Razon, O., et al. *arXiv*:2212.09470 (2022)

More results…

Novelty	Formula	Polynomials	Novelty	Formula	Polynomials
new and proven	$\frac{1+e}{-1+e}=2+\cfrac{1}{6+\cfrac{1}{10+\cfrac{1}{14+\cfrac{1}{\ddots}}}}$	$a_n=4n+2,\ b_n=1$	new and proven	$\frac{8}{-8+3\pi}=5+\cfrac{5}{7+\cfrac{12}{9+\cfrac{21}{11+\cfrac{32}{\ddots}}}}$	$a_n=5+2n,\ b_n=n(n+4)$
new and proven	$\frac{3}{3-e}=11-\cfrac{10}{29-\cfrac{28}{55-\cfrac{54}{89-\cfrac{88}{\ddots}}}}$	$a_n=2n(2n+7)+11,\ b_n=-2n(2n+3)$	new and proven	$\frac{4}{-2+\pi}=3+\cfrac{3}{5+\cfrac{8}{7+\cfrac{15}{9+\cfrac{24}{\ddots}}}}$	$a_n=3+2n,\ b_n=n(n+2)$
new and proven	$1+\frac{e}{e-2}=5-\cfrac{4}{19-\cfrac{18}{41-\cfrac{40}{71-\cfrac{70}{\ddots}}}}$	$a_n=2n(2n+5)+5,\ b_n=-2n(2n+1)+2$	known	$\frac{4}{\pi}=1+\cfrac{1}{3+\cfrac{4}{5+\cfrac{9}{7+\cfrac{16}{\ddots}}}}$	$a_n=1+2n,\ b_n=n^2$
new and proven	$\frac{e}{-24+9e}=6-\cfrac{1}{7-\cfrac{2}{8-\cfrac{3}{9-\cfrac{4}{\ddots}}}}$	$a_n=6+n,\ b_n=-n$	new and proven	$\frac{-4+3\pi}{20-6\pi}=5-\cfrac{2}{8-\cfrac{9}{11-\cfrac{20}{14-\cfrac{35}{\ddots}}}}$	$a_n=5+3n,\ b_n=-(n+1)(2n-1)$
new and proven	$\frac{e}{6-2e}=5-\cfrac{1}{6-\cfrac{2}{7-\cfrac{3}{8-\cfrac{4}{\ddots}}}}$	$a_n=5+n,\ b_n=-n$	new and proven	$\frac{4}{-8+3\pi}=3-\cfrac{1}{6-\cfrac{6}{9-\cfrac{15}{12-\cfrac{28}{\ddots}}}}$	$a_n=3+3n,\ b_n=-n(2n-1)$
new and proven	$\frac{1}{-16+6e}=3+\cfrac{1}{4+\cfrac{2}{5+\cfrac{3}{6+\cfrac{4}{\ddots}}}}$	$a_n=3+n,\ b_n=n$	new and proven	$\frac{8}{-8+3\pi}=6-\cfrac{3}{9-\cfrac{12}{12-\cfrac{25}{15-\cfrac{42}{\ddots}}}}$	$a_n=6+3n,\ b_n=-(n+2)(2n-1)$
new and unproven	$\frac{6e}{-3+2e}=7-\cfrac{4}{14-\cfrac{20}{23-\cfrac{54}{34-\cfrac{112}{\ddots}}}}$	$a_n=n(n+6)+7,\ b_n=-(n+3)n^2$	new and proven	$\frac{\pi}{4-\pi}=4-\cfrac{2}{7-\cfrac{9}{10-\cfrac{20}{13-\cfrac{35}{\ddots}}}}$	$a_n=4+3n,\ b_n=-(n+1)(2n-1)$
new and proven	$\frac{e}{-2+e}=4-\cfrac{1}{5-\cfrac{2}{6-\cfrac{3}{7-\cfrac{4}{\ddots}}}}$	$a_n=4+n,\ b_n=-n$	new and proven	$\frac{2}{10-3\pi}=4-\cfrac{3}{7-\cfrac{10}{10-\cfrac{21}{13-\cfrac{36}{\ddots}}}}$	$a_n=4+3n,\ b_n=-n(2n+1)$
new and proven	$\frac{1}{-5+2e}=2+\cfrac{1}{3+\cfrac{2}{4+\cfrac{3}{5+\cfrac{4}{\ddots}}}}$	$a_n=2+n,\ b_n=n$	new and proven	$\frac{2\pi+8}{\pi}=5-\cfrac{3}{8-\cfrac{12}{11-\cfrac{25}{14-\cfrac{42}{\ddots}}}}$	$a_n=5+3n,\ b_n=-(n+2)(2n-1)$
new and unproven	$\frac{3}{-10+4e}=3+\cfrac{4}{8+\cfrac{20}{15+\cfrac{54}{24+\cfrac{112}{\ddots}}}}$	$a_n=(n+1)(n+3),\ b_n=-(n+3)n^2$	new and proven	$\frac{2}{-2+\pi}=2-\cfrac{1}{5-\cfrac{6}{8-\cfrac{15}{11-\cfrac{28}{\ddots}}}}$	$a_n=2+3n,\ b_n=-n(2n-1)$
new and proven	$e=3-\cfrac{1}{4-\cfrac{2}{5-\cfrac{3}{6-\cfrac{4}{\ddots}}}}$	$a_n=3+n,\ b_n=-n$	new and proven	$\frac{6}{-8+3\pi}=5-\cfrac{5}{8-\cfrac{14}{11-\cfrac{27}{14-\cfrac{44}{\ddots}}}}$	$a_n=5+3n,\ b_n=-n(2n+3)$
new and proven	$\frac{1}{-2+e}=1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{\ddots}}}}$	$a_n=1+n,\ b_n=n$	new and proven	$1+\frac{\pi}{2}=3-\cfrac{2}{6-\cfrac{9}{9-\cfrac{20}{12-\cfrac{35}{\ddots}}}}$	$a_n=3+3n,\ b_n=-(n+1)(2n-1)$
known	$\frac{1}{-1+e}=\cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{\ddots}}}}$	$a_n=n,\ b_n=n$	new and proven	$\frac{2}{4-\pi}=3-\cfrac{3}{6-\cfrac{10}{9-\cfrac{21}{12-\cfrac{36}{\ddots}}}}$	$a_n=3+3n,\ b_n=-n(2n+1)$
new and proven	$\frac{e}{-1+e}=2-\cfrac{1}{3-\cfrac{2}{4-\cfrac{3}{5-\cfrac{4}{\ddots}}}}$	$a_n=2+n,\ b_n=-n$	new and proven	$\frac{2}{\pi}=1-\cfrac{1}{4-\cfrac{6}{7-\cfrac{15}{10-\cfrac{28}{\ddots}}}}$	$a_n=1+3n,\ b_n=-n(2n-1)$
new and unproven	$\frac{4e}{-1+2e}=3-\cfrac{3}{7-\cfrac{16}{13-\cfrac{45}{21-\cfrac{96}{\ddots}}}}$	$a_n=n(n+3)+3,\ b_n=-(n+2)n^2$	known	$\frac{4}{\pi}=1+\cfrac{1^2}{2+\cfrac{3^2}{2+\cfrac{5^2}{2+\dots}}}$	$a_n=2,\ b_n=(2n-1)^2$

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Factorial Reduction

	$\underline{n=1}$	$\underline{n=5}$	$\underline{n=10}$	$\underline{n=15}$	
$\frac{4}{\pi} = 1 + \cfrac{1^2}{3 + \cfrac{2^2}{5 + \cfrac{3^2}{7 + \ddots}}}$:	$\frac{4}{3}$	$\frac{29520}{23184}$	$\frac{100370793600}{78831037440}$	$\frac{3594206259195552000}{2822882994841190400}$	
Common divider:	1	144	5760	3628800	$\sim n!$
					"Factorial reduction"
Not related to fundamental constants $1 + \cfrac{1^2}{4 + \cfrac{2^2}{7 + \cfrac{3^2}{10 + \ddots}}}$:	$\frac{5}{4}$	$\frac{111339}{91140}$	$\frac{1666739432511}{1364364406260}$	$\frac{2714494483022211392 49}{2222038779020130353 40}$	
Common divider:	1	3	9	1053	
					No factorial reduction

Factorial Reduction-based Search

- Search for continued fractions with this new property
- Extremely rare in random continued fractions
- Appears in all historical examples
- Appears in all the examples that we found to express mathematical constants
- *Open question: Which continued fractions have factorial reduction? How to identify them in advance?*

Elimelech et al. in preparation

Factorial Reduction-based Search

α	a_n	b_n	**Formula**
0	$n^3+(n+1)^3$	$-n^6$	$\frac{1}{\zeta(3)}$
1	$n^3+(n+1)^3+4(2n+1)$	$-n^6$	$\frac{1}{1-\zeta(3)}$
2	$n^3+(n+1)^3+12(2n+1)$	$-n^6$	$\frac{8}{9-8\zeta(3)}$
3	$n^3+(n+1)^3+24(2n+1)$	$-n^6$	$\frac{216}{216\zeta(3)-251}$
...			
α	$n^3+(n+1)^3+2\alpha(\alpha+1)(2n+1)$	$-n^6$	$-\frac{2}{\psi^{(2)}(1+\alpha)}$

$$\frac{1}{\zeta(5)-\zeta(4)+\zeta(3)-\zeta(2)+1} = \lim_{n\to\infty} 2+\cfrac{-2}{49+\cfrac{-1536}{356+\cfrac{-78732}{\ddots+\cfrac{-n^{10}-n^9}{n^5+(n+1)^5+(n+1)^4}}}}.$$

$$\frac{9}{\Phi(1,3,\frac{8}{3})} = \frac{18}{26\zeta(3)-\frac{3591}{500}-\frac{4\pi^3}{3\sqrt{3}}} = \lim_{n\to\infty} 89+\cfrac{-81}{321+\cfrac{-5184}{715+\cfrac{-59049}{\ddots+\cfrac{-81n^6}{9\cdot(n^3+(n+1)^3)+80\cdot(2n+1)}}}}$$

$$\frac{64}{-273+64\zeta(3)+176\zeta(5)} = \lim_{n\to\infty} 13+\cfrac{-1}{165+\cfrac{-1024}{815+\cfrac{-59049}{\ddots+\cfrac{-n^{10}}{n^5+(n+1)^5+16\cdot(n^3+(n+1)^3)-4\cdot(2n+1)}}}}$$

Elimelech et al. in preparation

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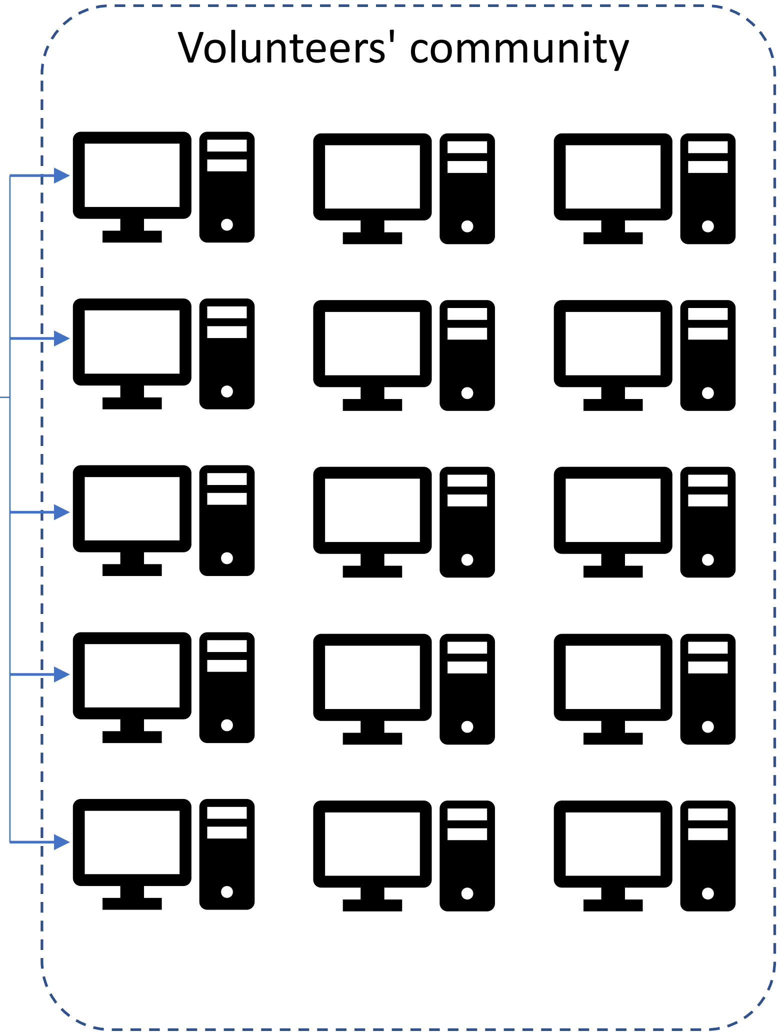
The Power of the People

Observation from previous results

Create a generalizing structure

Berkeley Open Infrastructure for Network Computing



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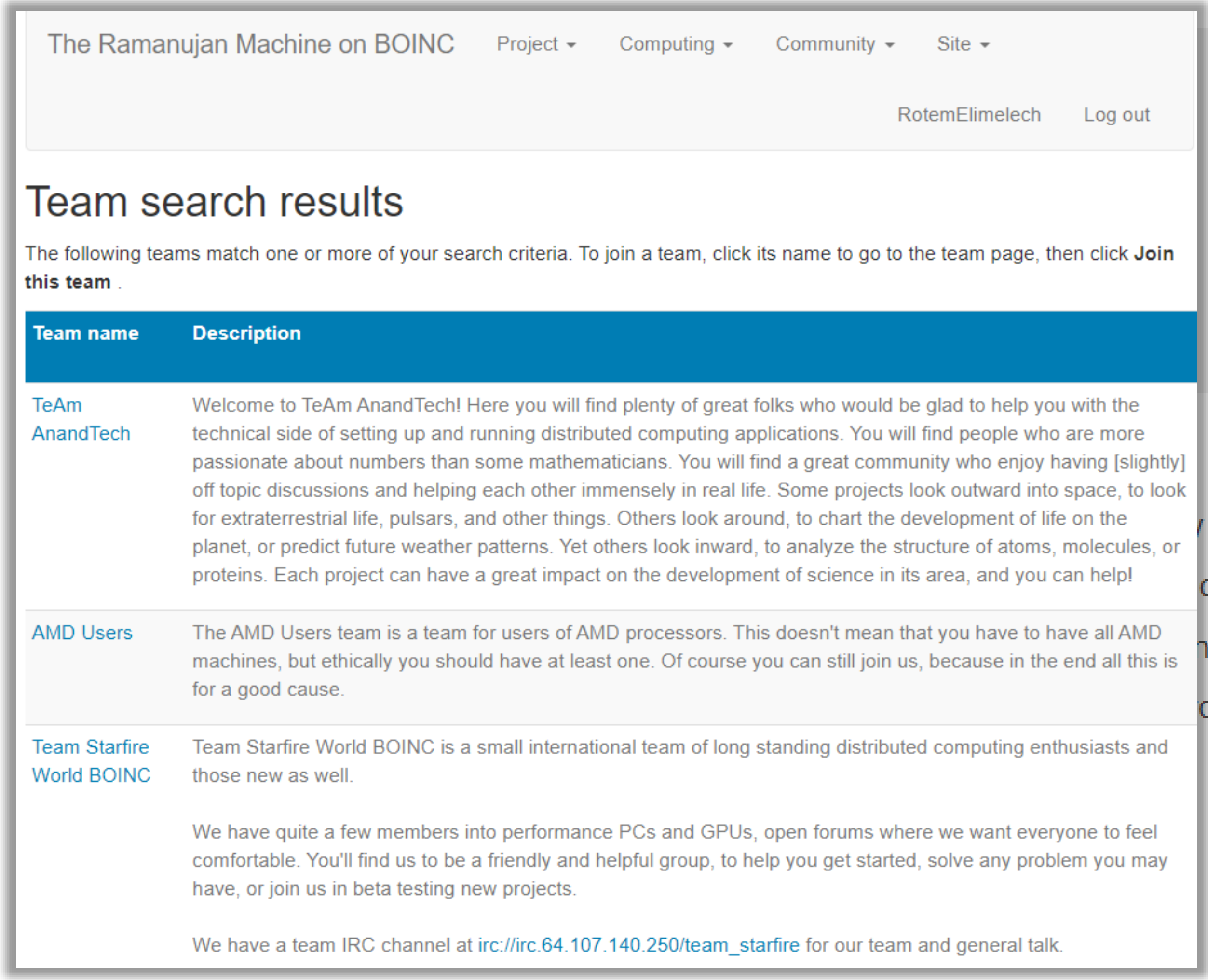
The Ramanujan Machine on BOINC   Project ▾   Computing ▾   Community ▾   Site ▾

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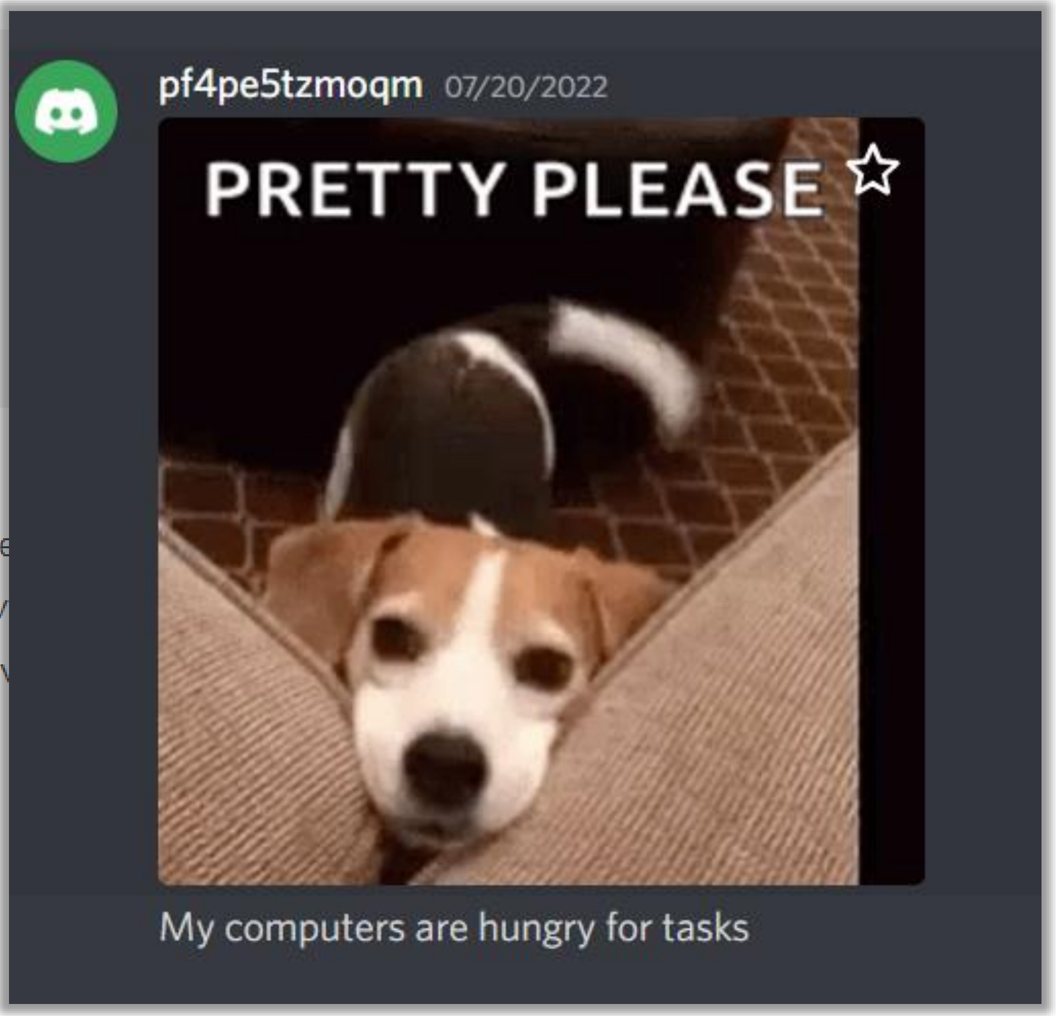
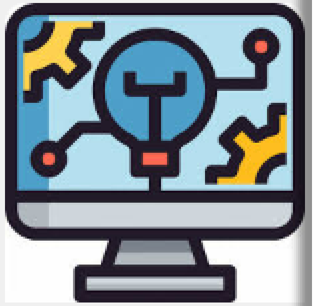
Team search results

The following teams match one or more of your search criteria. To join a team, click its name to go to the team page, then click **Join this team** .

Team name	Description
TeAm AnandTech	Welcome to TeAm AnandTech! Here you will find plenty of great folks who would be glad to help you with the technical side of setting up and running distributed computing applications. You will find people who are more passionate about numbers than some mathematicians. You will find a great community who enjoy having [slightly] off topic discussions and helping each other immensely in real life. Some projects look outward into space, to look for extraterrestrial life, pulsars, and other things. Others look around, to chart the development of life on the planet, or predict future weather patterns. Yet others look inward, to analyze the structure of atoms, molecules, or proteins. Each project can have a great impact on the development of science in its area, and you can help!
AMD Users	The AMD Users team is a team for users of AMD processors. This doesn't mean that you have to have all AMD machines, but ethically you should have at least one. Of course you can still join us, because in the end all this is for a good cause.
Team Starfire World BOINC	Team Starfire World BOINC is a small international team of long standing distributed computing enthusiasts and those new as well. We have quite a few members into performance PCs and GPUs, open forums where we want everyone to feel comfortable. You'll find us to be a friendly and helpful group, to help you get started, solve any problem you may have, or join us in beta testing new projects. We have a team IRC channel at irc://irc.64.107.140.250/team_starfire for our team and general talk.



Come join us!



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Zhu He (June 21st, 2020)

$$\frac{1}{-1+2\log(2)} = 3 - \cfrac{2}{6 - \cfrac{8}{9 - \cfrac{18}{12 - \cfrac{32}{15 - \cfrac{50}{\ldots}}}}} \quad (a(n) = 3n+3,\ b(n) = -2n^2 - 4n - 2)$$

Paul Michalski (July 18th, 2019)

$$\frac{4}{\pi} = 1 + \cfrac{1}{1 + \cfrac{4}{1 + \cfrac{3}{1 + \cfrac{8}{1 + \ldots}}}}$$

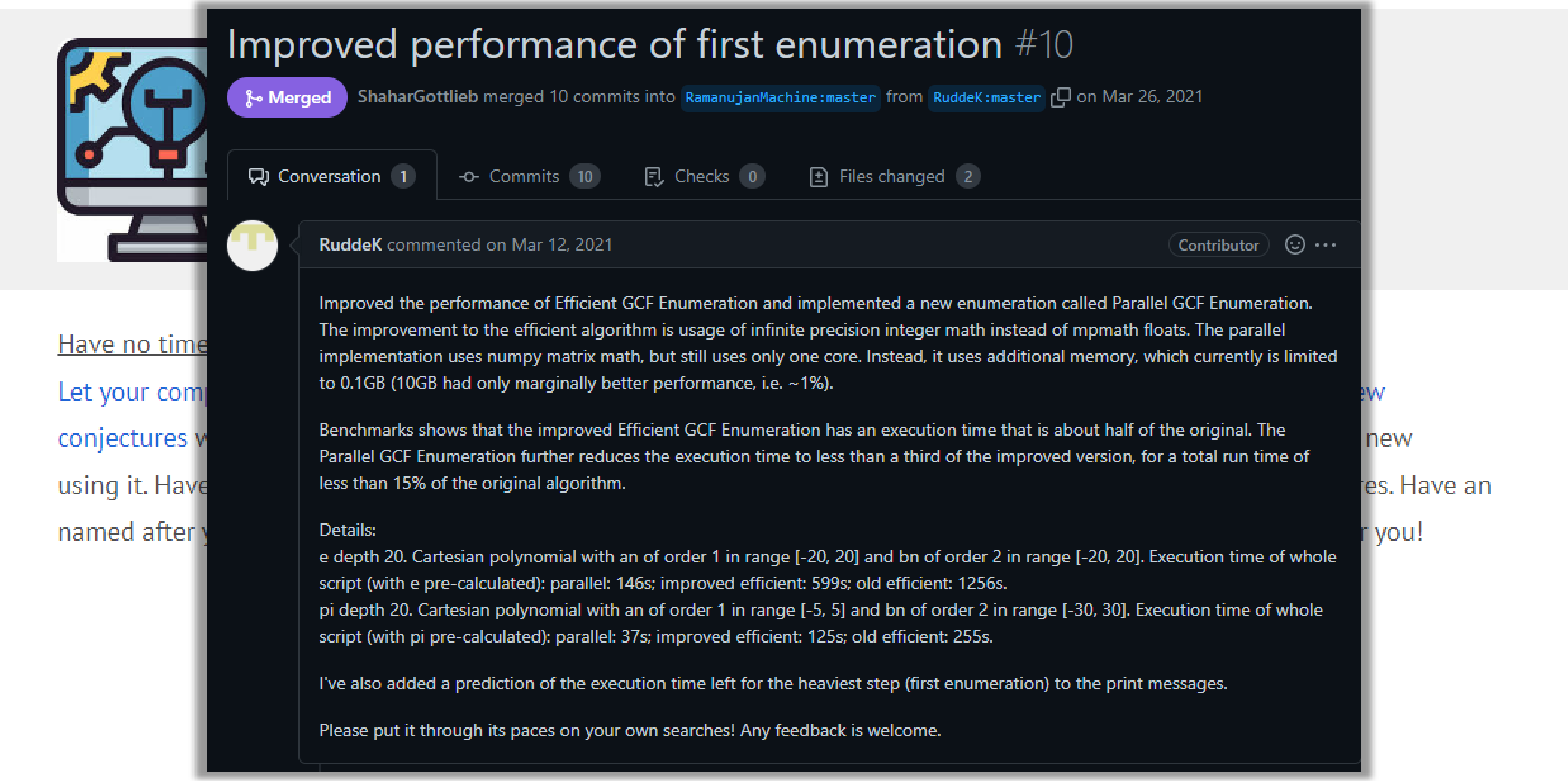
Éric Brier, David Naccache, and Ofer Yifrach-Stav

This team found an infinite family of continued fractions that generalizes many of the results from the first Ramanujan Machine paper. Below is a beautiful example from their work.

$$\frac{1}{\frac{25}{16} - \frac{9\pi \tanh\left(\frac{\pi\sqrt{7}}{2}\right)}{8\sqrt{7}}} = \frac{16}{3} - \cfrac{16}{23 - \cfrac{-256}{\frac{316}{5} - \cfrac{1764}{\ddots + \cfrac{-n^2(n^2+n+2)^2}{\frac{2n^4+11n^3+27n^2+36n+16}{n+3} + \ddots}}}}$$

Come join us!

www.ramanujanmachine.com

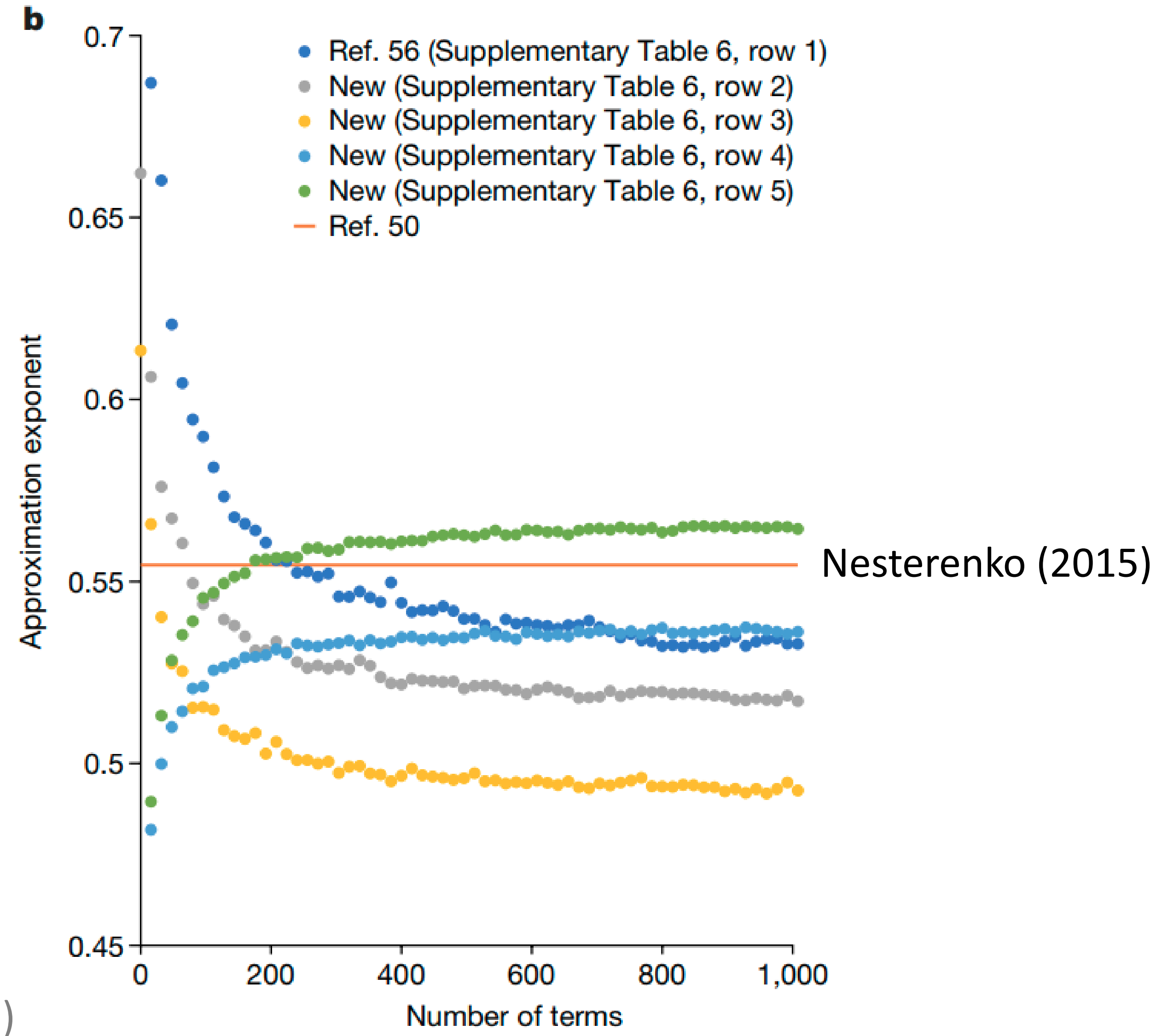


- The Goal: Automatic Generation of conjectures on Fundamental Constants
- Algorithms
- Distributed Computing Community
- **Mathematical Discoveries**
- Future Developments

New results in Irrationality Measure

$$G=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}$$

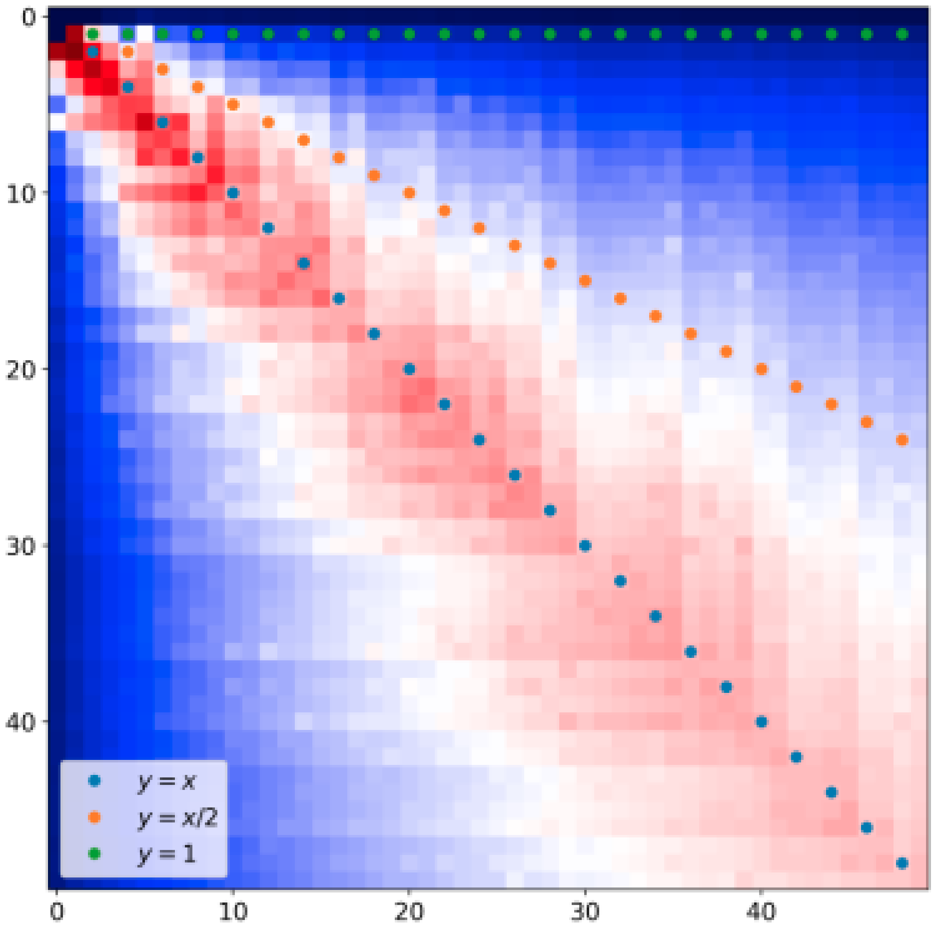
Constant	Polynomials	Rates
$-2565+\frac{-29}{2-2G}$	$a_n=-3520n^6-36608n^5-156944n^4-354816n^3-445924n^2-295296n-80503$ $b_n=16(20n^2+88n+97)(20n^2+8n+1)(2n+3)^4(n+1)^4$	$\frac{\text{digits}}{\text{term}}=2.089$ $\delta\approx 0.475$
$\frac{6837281250-33987530475}{-90G+83}$	$a_n=-2760376320n^{10}-50376867840n^9-410569703424n^8-1967378104320n^7-6136953513984n^6-13018455614976n^5-19015373884800n^4-18879847107648n^3-12191960479872n^2-4623075148716n-781542171975$ $b_n=768(9984n^4+92352n^3+319648n^2+490660n+281845)(9984n^4+12480n^3+5152n^2+804n+45)(6n+11)(6n+7)(4n+7)^2(4n+5)^2(2n+3)^2(n+1)^4$	$\frac{\text{digits}}{\text{term}}=3.432$ $\delta\approx 0.486$
$\frac{8379744738750G+7653030992325}{150G-137}$	$a_n=-173562407936n^{10}-3162164418560n^9-25716188549888n^8-122904265036800n^7-382183637133696n^6-807761798088192n^5-1174839578203360n^4-1160771163760576n^3-745418795117500n^2-280881243796872n-47150062304895$ $b_n=-27648(71248n^4+656848n^3+2263800n^2+3457460n+1974761)(71248n^4+86864n^3+32664n^2+4500n+225)(6n+11)^2(6n+7)^2(3n+5)(3n+4)(2n+3)^2(n+1)^4$	$\frac{\text{digits}}{\text{term}}=3.663$ $\delta\approx 0.51$
X_1	$a_n=C_1(n)*A_1(n+1)+C_1(n+1)*D_1(n)$ $b_n=-C_1(n-1)*C_1(n+1)*B_1(n)$	$\frac{\text{digits}}{\text{term}}=10.977$ $\delta\approx 0.462$
X_2	$a_n=C_2(n+1)*A_2(n+2)+C_2(n+2)*D_2(n+1)$ $b_n=-C_2(n)*C_2(n+2)*B_2(n+1)$	$\frac{\text{digits}}{\text{term}}=13.839$ $\delta\approx 0.4325$



Raayoni, G., Gottlieb, S., Manor, Y. et al. *Nature* **590,** 67–73 (2021)

Conserving Matrix Fields

α	a_n	b_n	**Formula**
0	$n^3+(n+1)^3$	$-n^6$	$\frac{1}{\zeta(3)}$
1	$n^3+(n+1)^3+4(2n+1)$	$-n^6$	$\frac{1}{1-\zeta(3)}$
2	$n^3+(n+1)^3+12(2n+1)$	$-n^6$	$\frac{8}{9-8\zeta(3)}$
3	$n^3+(n+1)^3+24(2n+1)$	$-n^6$	$\frac{216}{216\zeta(3)-251}$
...			
α	$n^3+(n+1)^3+2\alpha(\alpha+1)(2n+1)$	$-n^6$	$-\frac{2}{\psi^{(2)}(1+\alpha)}$



- Formal proof for irrationality of $\zeta(3)$!
- Systematic process
- Can be repeated to other constants

David et al. in preparation

Elimelech et al. in preparation

- The Goal: Automatic Generation of conjectures on Fundamental Constants
- Algorithms
- Distributed Computing Community
- Mathematical Discoveries
- **Future Developments**

Library of Integer Relations and Constants

	const_id [PK] uuid	value numeric	precision integer	time_added timestamp without time zone
1	03685cfc-...	114.9751467	68	2022-09-12 10:45:27.077479
2	060c0e44...	4.683887925	2000	2022-12-12 20:53:25.876545
3	09aa94ae...	0	2000	2022-09-12 10:45:37.565627
4	0b852d5c...	5.871178611	2000	2022-09-12 10:45:40.700258
5	0d96664c...	0	2000	2022-09-12 10:45:44.066509
6	0f885ed2...	6.511259917	2000	2022-12-12 20:53:41.786663
7	0faa0a9c-...	6.871757950	2000	2022-09-12 10:45:47.632681
8	119b05ae...	0	2000	2022-09-12 10:45:50.90368
9	137a490c...	7.872184629	2000	2022-09-12 10:45:54.047834
10	154a7536...	23.80970073	2000	2022-09-12 10:45:57.094241
11	172fffb0-...	0	4721	2022-09-12 10:46:00.266341
12	190c8114...	8.872511998	2000	2022-09-12 10:46:03.395289
13	1b1e5630...	4.991444235	2000	2022-09-12 10:46:06.845367
14	1d2eab46...	0	2994	2022-09-12 10:46:10.31178
15	1f43346e...	11.70459236	1999	2022-12-12 20:54:08.179041

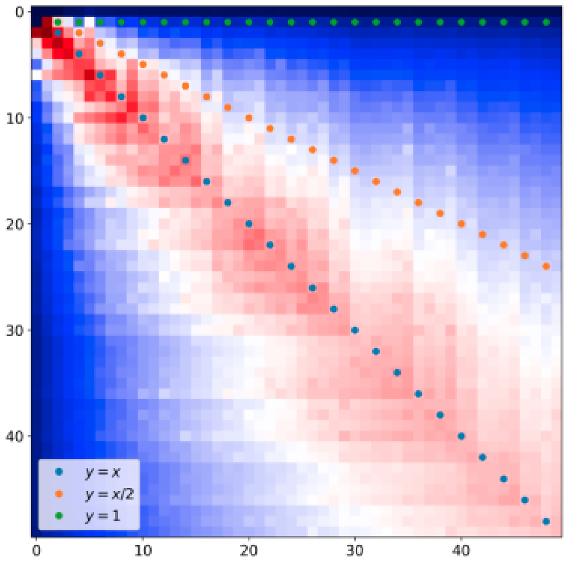
	relation_id [PK] uuid	relation_type character varying	details integer[]	time_added timestamp without time zone
1	0237827a-...	POLYNOMIAL_PSLQ	{2,1,0,-1,9,-2}	2022-12-21 03:32:52.991892
2	04198156-...	POLYNOMIAL_PSLQ	{2,1,0,-8,-1,3}	2022-12-21 01:04:43.807408
3	044af21a-...	POLYNOMIAL_PSLQ	{2,1,0,9,8,-5}	2023-01-26 14:30:44.152762
4	05fbd532-...	POLYNOMIAL_PSLQ	{2,1,0,-3,-4,1}	2023-01-24 20:59:42.129498
5	06f8c566-...	POLYNOMIAL_PSLQ	{2,1,0,-1,-6,1}	2023-01-24 20:31:05.797823
6	08f358d2-...	POLYNOMIAL_PSLQ	{2,1,0,6,-1,-1}	2023-01-28 12:05:19.70218
7	0ad1af80-...	POLYNOMIAL_PSLQ	{2,1,1,-8,0,0,4	2022-12-02 12:48:27.227349
8	0adf8a56-...	POLYNOMIAL_PSLQ	{2,1,0,-2,-1,1}	2022-12-20 23:15:10.226154
9	0cf7dd04-...	POLYNOMIAL_PSLQ	{2,1,0,1,1,-1}	2022-12-21 17:26:08.578565
10	0d0b777e-...	POLYNOMIAL_PSLQ	{2,1,0,-12,9,-1	2023-01-25 07:07:13.486252
11	0e2643ce-...	POLYNOMIAL_PSLQ	{1,1,0,-21,4}	2023-01-27 18:04:34.421882
12	0ed43370-...	POLYNOMIAL_PSLQ	{2,1,0,-1,-2,1}	2023-01-25 07:08:21.209485
13	0f88ee04-...	POLYNOMIAL_PSLQ	{2,1,1,0,0,-2}	2022-11-16 21:07:03.214841
14	13c42c48-...	POLYNOMIAL_PSLQ	{2,1,0,-10,5,1}	2023-01-26 20:29:04.93663
15	165ff506-...	POLYNOMIAL_PSLQ	{2,1,0,2,-1,1}	2023-01-26 13:05:20.529417

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0$$

- Publicly accessible
- Convenient interface

Beit-Halachmi et al. in preparation

Thank you!



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$$\frac{1}{\zeta(5)-\zeta(4)+\zeta(3)-\zeta(2)+1}$$

Raayoni, Gal, et al. "Generating conjectures on fundamental constants with the Ramanujan Machine." Nature 590.7844 (2021): 67-73.

Razon, Ofir, et al. "Automated Search for Conjectures on Mathematical Constants using Analysis of Integer Sequences." arXiv preprint arXiv:2212.09470 (2022).

Elimelech, Rotem, et al. "Algorithm-assisted discovery of a hierarchy in mathematical constants." In preparation

David, Ofir, et al. "The conservative matrix field." In preparation

Beit-Halachmi, Itay, et al. In preparation

www.ramanujanmachine.com

Irrationality criteria

Definition: The **irrationality measure** of $L \in \mathbb{R}$ is the largest δ for which

there exist a rational sequence $p_n/q_n \neq L$ s.t.

$$\left| L - \frac{p_n}{q_n} \right| < \frac{1}{q_n^{1+\delta}}.$$

Theorem (Roth's): The irrationality measure of L

$$\delta = \begin{cases} 0 & for\ L\ \boldsymbol{rational} \\ 1 & for\ L\ \boldsymbol{algebric\ irrational} \\ \geq 1 & for\ L\ \boldsymbol{trancdental} \end{cases}$$

Conclusion: If we find even one sequence with $\boldsymbol{\delta > 0 \Longrightarrow L}$ **is irrational**