Certified and parameterized latent space dynamics identification for time dependent image data

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Abstract

It is challenging to discover unknown governing equations in established scientific formalisms. It is especially so when the available data is high-dimensional, e.g., time-dependent image data. For example, classical symbolic regression techniques may not be the most suitable approach for several reasons, e.g., complex relationships between the input and output variables, combinatorial scaling, and high computational cost. To address this challenge, we compress the high-dimensional data onto a reduced latent space where a new set of generalized coordinates are defined. It is easier to identify unknown governing equations for the trajectory in the reduced latent space than the high dimensional space due to the reduced dimension. For example, one can fit the trajectory data in the reduced space into ordinary differential equations with a library of user-defined right-hand side functions through a regression technique, e.g., SINDy [1]. Here only the reduced number of variables are needed. The discovered ordinary differential equations are written in terms of the generalized coordinates, revealing how the generalized coordinates are interconnected to each other. Solving the identified governing equations in the reduced space can be done efficiently, whose solution can be easily mapped back to original high-dimensional image data. SINDy is just one way of identifying the latent space dynamics. We will discuss other possibilities as well, e.g., a fixed form of equations as in dynamic mode decomposition and thermodynamic formalism.

We call this framework *latent space dynamics identification*, so called LaSDI [2]. The LaSDI framework can be extended so that the discovered governing equations become *parametric*, where the coefficients of the user-defined functions are interpolated in terms of chosen parameters. This allows us to predict new time-dependent image data, completing the discovery procedure in a given parameter space. We use Guassian process interpolation, which provides the uncertainty of the LaSDI model in parameter space. The uncertainty can help achieve efficient image data generation because we can collect high dimensional image data at the maximum uncertainty point, which will minimize the overall uncertainty and improve the accuracy of the model. This sampling procedure can be thought of active learning framework, which ensures the minimal number of training data, but at the same time a desirable accuracy is certified.

We demonstrate LaSDI framework for several different two dimensional image data. First example is pore collapse hot spot image data, where a fixed form of linear dynamical system is used for latent space dynamics identification, i.e., dynamic mode decomposition. Here we demonstrate the efficiency of the LaSDI framework with data scarcity, whose performance is compared with a neural network model, i.e., recursive neural network. Second example is rising bubble image data where Gaussian process-based

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sampling procedure is compared with uniform sampling, demonstrating the uncertainty-based sampling outperforms a space-filling sampling method. The third example demonstrates that the LaSDI framework can be used for multi-query decision making process, i.e., optimal control of target temperature image reconstruction.

References

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