

## REDUCED ORDER MODELING AND DYNAMIC IDENTIFICATION FOR NONLINEAR PARAMETRIZED SYSTEM

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Most of the physical phenomena in nature can be mathematically described by nonlinear dynamical systems. Being able to discover parametric governing equations from scientific data is extremely significant, since it allows to represent dynamical features of the observed data – as well as to predict unobserved behaviors – such as regime transitions, variation in multiplicity and stability of solutions as input parameters change. This also proves essential in practical applications to avoid that, by varying parameters, the system might enter regimes that can compromise its operation like, e.g., the emergence of vorticity in fluid flows or resonance phenomena in mechanical structures.

We present a data-driven, non-intrusive framework which breaks down the discovery problem into two interacting subtasks: the identification of a change of variables, which reduces dimensionality and provides more suitable coordinates to describe the dynamical phenomenon, and the effective identification of the dynamics in the new coordinate system. By providing as input the time evolutions of the states of the system for few parameter configurations, the proposed approach leverages autoencoder neural networks and parametric sparse identification of nonlinear dynamics (SINDy) to create a low-dimensional dynamical model. This model can be used to efficiently estimate solutions at new parameter instances, as well as directly fed to continuation algorithms. These aim at tracking the evolution of periodic steady-state responses as function of parameters, thus allowing to detect instabilities and bifurcations.

This framework represents a flexible tool for combining the expressiveness of neural networks with physical knowledge of the phenomenon. In fact, prior knowledge can be informed to guide the choice of possible candidate models in the system identification phase; conversely, when no a priori information is at disposal, the new coordinates identified by the autoencoder can unveil dynamic features and, ultimately, normal forms of the dynamics of the observed phenomena.

Effectiveness of the method is illustrated on two very diverse examples in fluid dynamics and structural mechanics. The first example deals with the hardening behavior of a beam structure, while the second one considers the transition of a fluid flow from laminar to unsteady behavior.

### REFERENCES

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